Downloaded from buildwithruud.com · Kami Document System

An Exploratory Diagonalization Argument Concerning P

2026.11.16 · /blog/exploratory-diagonalization-argument-p-vs-np

Important Disclaimer

This work is an exploratory mathematical paper. While every step follows logically from the stated definitions and constructions, the P vs NP problem remains an open question in theoretical computer science. The argument presented here is offered in the spirit of rigorous inquiry into the boundary between Cook–Levin tableau reductions and self-referential fixed points.

1. Definitions

Class P**

A language $L \subseteq \{0,1\}^*$ belongs to **P** if there exists a deterministic Turing machine M and a constant $c \geq 1$ such that for every input x with $|x| = n$, M decides membership of x in L and halts within n^c steps.

Class NP**

A language $L \subseteq \{0,1\}^*$ belongs to **NP** if there exists a deterministic Turing machine V (the verifier) and a constant $c \geq 1$ such that:

V runs in at most n^c steps on inputs of the form (x, w) where $|x| = n$;

$x \in L$ if and only if there exists a witness string w with $|w| \leq n^c$ for which V accepts.

2. Key Constructions

3-SAT belongs to NP**

The language 3-SAT consists of the encodings of satisfiable 3-CNF formulas. A satisfying assignment serves as a witness of polynomial length. Verification consists of substituting the assignment into each clause and checking truth values, which can be performed in linear time. Thus 3-SAT is in NP.

Cook–Levin Construction**

Let $L \in \mathbf{NP}$ with verifier V running in time $O(n^c)$. For any input x of length n, one can construct in polynomial time a 3-CNF formula ψ_x of size $O(n^{O(c)})$ such that $x \in L$ if and only if ψ_x is satisfiable.

The construction proceeds by creating Boolean variables for each cell of a computation tableau of size $O(t(n)) \times O(t(n))$, where $t(n) = O(n^c)$. Clauses are added to enforce:

Local consistency with the finite transition function of V;

Correct initial configuration containing the input x;

Acceptance in the final time step.

This yields a polynomial-size 3-CNF whose satisfiability is equivalent to membership in L. If 3-SAT admits a polynomial-time algorithm, then every language in NP would also admit a polynomial-time algorithm by composition with the Cook–Levin reduction.

3. Exploratory Diagonalization Argument

We now examine what follows from the hypothesis that $\text{3-SAT} \in \mathbf{P}$.

Assume there exists a deterministic Turing machine A and constant $k \geq 1$ that correctly decides any 3-CNF formula of size n in at most $O(n^k)$ steps.